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Abstracts

Long memory in option pricing: A fractional discrete-time approach

Jean François Bégin

Simon Fraser University

Abstract: In this presentation, we study the impact of long memory on volatility modelling and option pricing. We propose a general discrete-time pricing framework based on affine multi-component volatility models that admit ARCH(∞) representations. This not only nests a large variety of option pricing models from the literature, but also allows for the introduction of novel covariance-stationary long-memory affine GARCH models. Using an infinite sum characterization of the log-asset price's cumulant generating function, we derive semi-explicit expressions for the valuation of European-style derivatives under a general variance-dependent stochastic discount factor. Moreover, we carry out an extensive empirical analysis using returns and cross sections of S&P 500 options over the period 1996 to 2019. Overall, we find that once the informational content from options is incorporated into the parameter estimation process, the inclusion of long-memory dynamics in volatility is beneficial for improving the out-of-sample option pricing performance. The largest improvements in the implied volatility root-mean-square errors generated by our long-memory models occur for options with maturities longer than one year, reaching 33% and 13% when compared to one- and two-component short-memory models, respectively.

Galerkin Approach for Spread Contract Valuation

Ciro Díaz (joint work with Pablo Olivares)

Toronto Metropolitan University

Abstract: We study two efficient approaches to value spread options on commodities whose underlying assets follow a dynamic described by two-dimensional Levy models, by solving their associated partial integro-differential equation (PIDE). To this end we consider a Galerkin discretization in space along with an implicit θ-scheme for time evolution. Diffusion and drift in the associated operator are discretized using an exact Gaussian quadrature. The integral part corresponding to jumps is approximated using the *symbol method* when the symbol is available, or using exact quadrature when the pdf is available. A system with blocked Toeplitz with Toeplitz blocks (BTTB) matrix is efficiently solved via biconjugate stabilized gradient method (BICSTAB) with a circulant pre-conditioner at each time step.

Metamodeling for Variable Annuity Valuations: 10 Years Beyond Kriging

Guojun Gan

University of Connecticut

Abstract: Variable annuities are retirement insurance products created by insurance companies that contain financial guarantees. To mitigate the financial risks associated with these guarantees, insurance companies have adopted dynamic hedging, which is a risk management technique for options. However, dynamic hedging is associated with computationally intensive valuations of variable annuity policies. Recently, metamodeling approaches have been developed to address the computational problems. A typical metamodeling approach consists of two components: an experimental design method and a metamodel. In this talk, we will give a review of metamodeling approaches developed in the past ten years.

Equal Risk Pricing of Derivatives with Reinforcement Learning

Frédéric Godin

Concordia University

Abstract: The equal risk pricing methodology for derivatives pricing is introduced. The deep reinforcement learning associated implementation is discussed. Numerical experiments results are presented, along with an analysis of the choice of the objective function and of the hedging instruments. The approach is also benchmarked against traditional pricing methods.
The talk is based among others on the following papers:

Carbonneau, A., & Godin, F. (2021). Equal risk pricing of derivatives with deep hedging. Quantitative Finance, 21(4), 593-608.

Carbonneau, A., & Godin, F. (2021). Deep equal risk pricing of financial derivatives with multiple hedging instruments. arXiv preprint arXiv:2102.12694.

Carbonneau, A., & Godin, F. (2021). Deep equal risk pricing of financial derivatives with non-translation invariant risk measures. arXiv preprint arXiv:2107.11340.

The Hurst roughness exponent and its model-free estimation

Xiyue Han (joint work with Alexander Schied)

University of Waterloo

Abstract: We say that a continuous real-valued function x admits the Hurst roughness exponent H if the p^{th} variation of x converges to zero if $p > 1/H$ and to infinity if $p < 1/H$. For the sample paths of many stochastic processes, such as fractional Brownian motion, the Hurst roughness exponent exists and equals the standard Hurst parameter. In our main result, we provide a mild condition on the Faber–Schauder coefficients of x under which the Hurst roughness exponent exists and is given as the limit of the classical Gladyshev estimates $\hat{H}_n(x)$. This result can be viewed as a strong consistency result for the Gladyshev estimators in an entirely model-free setting, because no assumption whatsoever is made on the possible dynamics of the function x. Nonetheless, our proof is probabilistic and relies on a martingale that is hidden in the Faber–Schauder expansion of x. Since the Gladyshev estimators are not scale-invariant, we construct several scale-invariant estimators that are derived from the sequence $(\hat{H}_n)_{n\in\mathbb{N}}$. We also discuss how a dynamic change in the Hurst roughness parameter of a time series can be detected. Finally, we extend our results to the case in which the p^{th} variation of x is defined over a sequence of unequally spaced partitions. Our results are illustrated by means of high-frequency financial time series.

Convolution-FFT for option pricing in the Heston model

Cody Hyndman (joint work with Xiang Gao)

Concordia University

Abstract: In this paper we first provide an analytic expression for the joint characteristic function of the log-price and variance in the Heston model that addresses the discontinuity problem. We then consider option valuation using the fast Fourier transform (FFT) and convolution. We apply shifting and damping transforms to improve boundary errors and prove an error estimate. Finally, numerical experiments and comparisons to Carr and Madan (1999) illustrate the speed and accuracy of our approach.

Credit Risk Modelling with Occupation Times via Spectral Expansions

Hiromichi Kato

Willfrid Laurier University

Abstract: There are two main branches of mathematical models: structural and reduced-form models. Structural models assume defaults trigger endogenously with the firm value. The Black–Cox model is the first structural model to consider early defaults by forced liquidation of assets. Broadie et al. proposed a model, allowing the firm to restructure its debt. Reduced-form models are intensity-based models in which the likelihood of a default is measured by a hazard rate process. Reduced-form models lack the ability to determine defaults endogenously with the firm value. Alfonsi et al. proposed a hybrid model that unifies the Black–Cox model and reduce-form models, where the hazard rate process is driven endogenously by the firm value and other exogenous factors.

Models proposed by Broadie et al. and Alfonsi et al. are occupation time-based models and assume the dynamics of the firm value follow a geometric Brownian motion (GBM). I will propose an alternative approach by the spectral expansion methodology. Spectral expansions are used to price occupation time derivatives, such as step options. I will show how the spectral expansion approach can be utilized in the Broadie et al. and the Alfonsi et at. models. Some analytical results and numerical illustrations, pertaining to the proposed models, will be presented.

Monte Carlo and Quasi-Monte Carlo Methods with Applications in Financial Engineering

George Lai

Wilfrid Laurier University

Abstract: The Monte Carlo simulation method is indispensable to deal with high-dimensional problems. It is widely used in many fields. The most recent application fields of this method are artificial intelligence and big data. The main drawback of this method is the issue of slow convergence. To accelerate the convergence, variance reduction methods, effective dimension method, quasi-Monte Carlo methods, and their combinations were developed. In this talk, we will introduce a speeding-up method of the Monte Carlo and quasi-Monte Carlo simulation method with appli- cations to derivative pricing under an exponential time-changed or subordinated Brownian motion process model.

Batch mode active learning framework and its application on variable annuity portfolio valuation

Shu Li

Western University

Abstract: In practice, the valuation of a large volume variable annuity contracts relies on Monte Carlo simulation which is computationally intensive. To build a more efficient valuation process, metamodeling approaches have been used that consist of two subsequent stages: the data sampling stage to create a set of representative contracts, and the regression modeling stage to make predictions for the remaining contracts in the portfolio. In this talk, I will focus on a new data mining framework based on active learning, in which we iteratively update the regression model efficiently by expanding the representative data with the most informative contracts. Our metrics take into consideration both the ambiguity and the diversity of the prediction, where two random sampling algorithms are proposed based on random forest. Experimental results demonstrate the effectiveness of the proposed approaches over the random sampling as well as the two-stage metamodeling framework.

Optimal stopping with discontinuous and time-dependent reward and applications to variable annuities

Anne MacKay

Université de Sherbrooke

Abstract: We consider a financial derivative with early exercise whose reward function is time-dependent and presents a discontinuity at maturity. In this context, the regularity conditions required to apply the results and techniques used in the American option literature are not satisfied. Using more general results on optimal stopping for continuous processes, we confirm the existence of an optimal stopping time for our problem and study the regularity of the resulting value function. This allows us to express the price of our derivative in terms of a free boundary problem and to apply methods similar to those developed for American options. We finally present an application of our results to the valuation of variable annuities with early surrenders.

Pricing Multi-Asset Options under a Jump-Diffusion Model with a Systemic Risk Component

Roman Makarov

Wilfrid Laurier University

Abstract: A jump-diffusion asset-price model, where a common "systemic risk" security is combined with several conditionally independent "ordinary" assets, has been recently proposed in [1] and [2]. This approach allows for analyzing a portfolio that integrates a high-activity security, such as an ETF tracking a major market index, and several low-activity securities. The latter may have missing or asynchronous pricing data due to infrequent trading on financial markets. Thus, one of the significant features of the proposed model is the possibility of estimating parameters for each asset price process individually, even if historical pricing data are unavailable for every period. It can be achieved by employing a conditional maximum likelihood estimation (MLE) method. We can also use empirical single-asset option data to calibrate the model under the risk-neutral probability measure. Note that the number of parameters grows linearly in the number of ordinary assets compared to the quadratic growth through the correlation matrix, which is typical for a general multi-asset price model. An essential property of the proposed model is that the asset price processes are conditionally independent given the systemic-risk asset value. By conditioning on the systemic-risk asset, we can price basket European-style options and compute risk measures such as VaR and CVaR.

In this talk, we present deterministic methods for pricing European-style basket options and consider three examples: geometric average, maximum price, and arithmetic average. For pricing a call or put option on the geometric average of asset prices, we find a characteristic function of the average and use the Carr–Madan method. For evaluating a basket option with a maximum (or minimum) asset price, we find the probability distribution of the max/min price given the common factor value and then compute the price as an iterated integral. Lastly, to find the value of a call or put option on the arithmetic average, we combine the Laplace transform and change of numeraire approach with the approximation for a sum of independent lognormal random variables proposed in [3]. We compare the semi-analytical methods with Monte Carlo results. Additionally, in this talk, we discuss some ways to extend the proposed model by using a Hawkes process for jump dynamics, a square-root diffusion for stochastic volatility, and a Jacobi process for random correlations.

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3. Furman E., Hackmann D., and Kuznetsov A. On log-normal convolutions: An analytical–numerical method with applications to economic capital determination. Insurance: Mathematics and Economics, 90:120–134, 2020.

Market-making Options with Illiquid Underliers

Christian Maxwell

Western University

Abstract: In a complete market, there is a unique, no-arbitrage, price for every derivative contract. However, all markets are incomplete to some degree, if only because liquidity costs apply for all trades. We propose a model of market impact and transaction costs to reflect order book liquidity, and develop a utility indifference formulation to compute the maker's bid/ask prices along with the associated optimal hedging strategies. We then use this model to numerically illustrate how market frictions in the hedge asset affect derivative value by solving the associated HJB equations while developing market intuition from the resulting prices and hedge ratios.

On ruin probability estimation for optional processes

Alexander Melnikov

University of Alberta

Abstract: Risk theory is concerned with the study of stochastic models of risk in finance and insurance, and its central problem is identified with the ruin probability estimation. In the talk we show how methods of optional processes work in this area. In our setting the evolution of the capital of insurance company is described as an optional semimartingale on a stochastic basis without so-called "usual conditions". The optional risk process may admit jumps from both left and right sides at any time. We study the ruin probability estimation developing an extended technique of stochastic exponentials in this very general setting. We prove that the ruin probability for the optional risk process admits exponential upper bounds. We show that many well-known models and estimates in risk theory can be derived from our main results. Moreover, we provide a reasonable motivation to use optional processes in this area and give several illustrative examples. The talk is based on the paper "An optional semimartingales approach to risk theory" by Mohamed Abdelghani, Alexander Melnikov and Andrey Pak (2022).

Enhancing Mortality Prediction via Borrowing Information

Yechao Meng

University of Waterloo

Abstract: It has been widely recognized that populations share certain types of similarities in their mortality development patterns across populations and ages. Making full use of the information hidden in human mortality data can be conductive to enhance the predicting accuracy of future mortality. We designed effective statistical learning-based frameworks for borrowing the right amount of information from different angles to assist the mortality prediction of a predicting target and investigate their performance with real data study.

Pricing Bitcoin Derivatives under Jump-Diffusion Models

Pablo Olivares

Toronto Metropolitan University

Abstract: In recent years cryptocurrency trading has captured the attention of practitioners and academics. The volume of the exchange with standard currencies has known a dramatic increasing of late. This talk addresses to the need of models describing a bitcoin-US dollar exchange dynamic and their use to evaluate European option having bitcoin as underlying asset.

Introduction to Switching Loss Distribution for Climate Disasters: A Case Study of United States Climate Disaster Losses

Ali Raisolsadat (joint work with Kai Liu)

University of Prince Edward Island

Abstract: Since the late 20th century, our world has experienced extreme weather events accompanied by minor to extensive losses to individuals and communities. In 2015, the United Nations proposed a resolution, the Paris Agreement, to enforce and help nations mitigate and adapt to the changes in the earth's climate. Since then, the signatory parties have implemented plans to increase their resilience and set sustainability goals to further mitigate and prevent extreme events from happening in the future. While the signatory countries are busy creating plans and policies for mitigation, the scientific community needs to provide policymakers with adaptation options for severe climate events and their physical impacts. There is a significant concern that privatized companies are neither accurately reporting their risk nor preparing for climate change physical impacts. Therefore, by studying the United States weather-climate billion-dollar disasters since 1980, we decided to construct an accurate general loss distribution, called Switching Loss Distribution, to provide a helpful model for policymakers and further motivate the financial and insurance industry to consider climate change risk planning. For this study, our goal was not only to show how simple it is to consider the impacts of climate change but to construct a loss distribution that (1) would have the ability to differentiate losses between a climate disaster losses and climate catastrophe losses, (2) would be simple to understand (naive approach), (3) have robust for parameter selection, and (4) can be adapted to different scenarios of climate change loss modeling. After creating the Switching Loss Distribution, we showed how simple the parameter selection is and how well the model performed for its robustness.

Optimal Pricing of Climate Risk

Alexey Rubtsov

Toronto Metropolitan University

Abstract: The climate change model of Daniel et al. (Proc Natl Acad Sci USA 116(42):20886-20891, 2019. https://doi.org/10.1073/pnas.1817444116) is an important contribution to the carbon pricing literature. However, the computational methodology proposed in this paper is costly, thus limiting the flexibility and scalability of this basic approach. In this paper we introduce several modern computational techniques, often used in optimization applications, such as vectorization and automatic differentiation for gradient computation, to dramatically improve computational performance: this allows for increased scalability and sensitivity analysis (including the analysis of suboptimal policies). Such studies are reported in this paper yielding a number of new experimental insights. For example, our new code illustrates the cost of postponing climate change mitigation is significant. Specifically, postponing climate change mitigation until 2030 is equivalent to giving up $5.4 trillion (in 2015 US dollars).
