Assignment #1

Date Due: November 2, 2021

Total: 100 marks

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1. (20 marks) Compute the languages

   $L_1 = \{$the set of all strings over the alphabet $\{0, 1\}$ that begin with 1010$\}$

   $L_2 = \{$the set of all strings over the alphabet $\{0, 1\}$ that end with 0101$\}$

   $L_3 = \{$the set of all strings over the alphabet $\{0, 1\}$ that begin with 00110$\}$

   For each of the languages

   (a) $L_4 = L_1 \cap L_2$, and
   (b) $L_5 = L_3 \cap L_2$

   give a DFA for accepting it over the alphabet $\{0, 1\}$.

2. (40 marks) Compute the following languages over the alphabet $\{0, 1\}$:

   (a) the set of all strings consisting of alternating groups of 11 and 101 (11 and 101 *alternates* at least once);
   (b) the set of all strings whose forth symbol from the right end is an 1;
   (c) the set of strings that either begin, or end (or both) with 0101;
   (d) the set of strings such that the number of 0's is divisible by six, and the number of 1's is not divisible by seven.

   and for each case give a DFA accepting it.

3. (20 marks) Give DFA's accepting the following languages over the alphabet $\Sigma = \{0, 2, 3, 5, 7\}$:

   (a) the set of all strings beginning with a 7, 3 or 5, that, when the string is interpreted as an integer in base 9, is a multiple of 5 plus 2. For example:
   - strings 7, 30, 35, 52, 502, 5002, 5057, 50057, 705, and 77777 are in the language;
   - the strings 20, 00,022, 0020, 37 , 23, 33, 223, 2325, 2375, 3, 5, 33333,, 22222, 505, 22, 72, and 035 are not.

   (b) The set of all strings that ends with an **7, 3, or 5** and when the string is interpreted ***in reverse*** as an integer **in base 9, is a multiple of 5 plus 2**.

   Examples of strings in the language are 7, 03, 53, 25, 205, 2005, 7505, 75005, 507, and 77777 Examples of strings that are not in the language are: 02, 00,220, 0200, 73 , 32, 33, 322, 5232, 5732, 3, 5, 33333,, 22222, 505, 22, 27, and 530.

4. (20 marks) Let $p \in \mathbb{N}$, p prime, $p > 4$, and $h \in \mathbb{N}$ such that $1 \leq h < p$. We have a DFA $A = (\Sigma, Q, \delta, 0, F)$ with $Q = \{0, 1, \ldots, k\}$, $k \geq p$, $\{a, b, c\} \subseteq \Sigma$. We have that $\delta(q, a) = q + (p-3)h \bmod p$, for all states $q \in Q$. In these conditions:
   (a) show by induction on n that for all $n \geq 0$ and $q < p$, $\overline{\delta}(q, a^{n \cdot p}) = q$;
   (b) show that either $\{a^p\}^* acba^p abac \subseteq L(A)$, or $\{a^p\}^* acba^p abac \cap L(A) = \emptyset$.

5. (10 marks) Consider the DFA with the following transition table:

	0	1
→ 0	1	0
1	2	1
2	3	2
∗ 3	1	3

   Informally describe the language accepted by this DFA, and prove that your description is correct. You may use a proof based on induction on the length of an input string.

6. (10 marks) Repeat the above exercise for the following transition table:

	0	1
→ A	D	A
∗ B	C	B
C	B	C
D	A	D

The maximum is bounded to 115 marks.

Very Important: Verify your solutions using Grail; describe ***how do you think*** for each of the above exercises. Just giving the final solution without any explanation may result in a mark of 0 at the discretion of your instructor.

If you decide for a late submission, please, contact me, before the due date, because I will give the solutions to ***all*** exercises in class.