Assignment #2

Date Due: November 6, 2024

Total: 100 marks

We have the following languages:

$L_1 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ that begin with 1010$\}$,
$L_2 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ that end with 01011$\}$,
$L_3 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ having 010 a a subword$\}$,
$L_4 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ having an odd number of 0's$\}$,
$L_5 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ having an even number of ~~1~~2's$\}$,
$L_6 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ having the ~~fifth~~ forth symbol from the right end a 1$\}$,
$L_7 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ beginning with ~~22021~~12022$\}$,
$L_8 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ ending in ~~22021~~12022$\}$,
$L_9 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ having the number of 1's multiple of 6$\}$,
$L_{10} = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ having the number of 1's multiple of 5$\}$,
$L_{11} = \{$the set of all strings over the alphabet $\{a, b\}$ having the number of a's multiple of 7$\}$,
$L_{12} = \{$the set of all strings over the alphabet $\{a, b\}$ having the number of b's multiple of ~~5~~6$\}$,
$L_{13} = \{$the set of all strings consisting of alternating groups of ~~12~~10 and ~~02~~01 (~~12~~10 and ~~02~~01 *alternates* at least once)$\}$,

and the following homomorphisms

$h : \{a, b\} \longrightarrow \{0, 1, 2\}^*$, $g : \{0, 1, 2\} \longrightarrow \{a, b\}^*$, $h(a) = 01$, $h(b) = 21$, $g(0) = a$, $g(1) = ba$.

We also have the following languages computed in Assignment #1:

1. $L_{20} = L_1 \cap L_2$.
2. $L_{21} = 01011\Sigma^* \cap \Sigma^*1010$
3. $L_{22} = L_{13}$
4. $L_{23} = L_6$
5. $L_{24} = L_7 \cap L_8$
6. $L_{25} = L_{11} \setminus L_{12}$
7. $L_{26} = h^{-1}(L_4)$
8. $L_{27} = h^{-1}(L_1^R) \cap h^{-1}(L_5)$
9. $L_{28} = g(L_1^R)$

1. (60 marks) For each of the following languages give a regular expression generating them over the alphabet $\{0, 1, 2\}$ or $\{a, b, c\}$, depending on the description of the language (10 marks each):

   (a) L_{20}

   (b) L_{21}

   (c) L_{22}

   (d) L_{23}

   (e) L_{24}

   (f) L_{25}

   (g) L_{26}

   (h) L_{27}

   (i) L_{28}

2. (20 marks) Write regular expressions for the following languages over the alphabet $\Sigma = \{0, 1, 2, \cancel{3}, 4, \cancel{5}, 6\}$:

   (a) the set of all strings beginning with a **1, ~~3~~2 or ~~5~~4**, that, when the string is interpreted as an integer **in base ~~9~~7, is a multiple of ~~5~~4 plus ~~2~~1**. For example:

   - strings ~~13,30,35,1333,1316,1613,513,563, and 55563~~ 1,41,210,221,2061,2010, 2612, 202012,102642, and 440614 are in the language;
   - the strings ~~2,3,5,135,136,20, 00, 022, 0020, 37 , 23, 5057, 223, 2325, 2375, 32222, 505, 22, 72, and 035 are not.~~ 2, 4, 01, 21, 212, 610, 0221, 4062,4021,6014, and 035 are not.

   (b) The set of all strings that ends with an **1, ~~3~~2, or ~~5~~4** and when the string is interpreted ***in reverse*** as an integer **in base ~~9~~7, is a multiple of ~~5~~4 plus ~~2~~1**.

   - Examples of strings in the language are ~~31, 03, 53, 3331, 6131, 3161, 315, 365, and 36555~~ 1,14,012,122,1602,0102,2162, 210202, 246201, and 416044.
   - Examples of strings that are not in the language are: ~~2,3,5,531,631,02, 00, 220, 0200, 73 , 32, 7505, 322, 5232, 5732, 22223, 505, 22, 27, and 530.~~ 2, 4, 10, 12, 212, 016, 1220, 2604 , 1204, 4106 , and 530.

3. (25 marks) Consider the DFA with the following transition table:

	0	1
$\rightarrow 0$	1	0
1	2	1
$* 2$	3	2
3	1	3

(a) (10 marks) Find the equivalent regular expression using the algorithms learned in class.

(b) (10 marks) Transform the regular expression into an ε-NFA

(c) (10 marks) Transform the ε-NFA into a DFA.

4. (25 marks) Check your results with Grail+ and comment on the Grail+ experiments (another 5 marks/test(language)).