Assignment #4

Date Due: December 2, 2024

Total: 100 marks

Use only the algorithms from the slides/notes I provided for this assignment.

1. (10 marks) Write a context-free grammar that generates the language

$$\{w \in \{a,b\}^* \mid |w|_a = |w|_b - 1\}.$$

2. (10 marks) Prove that the following grammar is ambiguous

   $S \to A\%B|B\%A,\ \ A \to A * A|a|b,\ \ B \to a|b|B * A.$

3. (15 marks) Write an equivalent ***REGULAR*** grammar, ***in canonical form***, for the following DFA

```
(START) |- 0        4 a 3
0 a 2               4 b 2
0 b 1               5 a 6
1 a 0               5 b 4
1 b 6               6 b 7
2 b 5               6 a 5
2 a 3               7 a 7
3 a 7               7 b 7
3 b 6               0 -| (FINAL)
                    1 -| (FINAL)
                    4 -| (FINAL)
                    6 -| (FINAL)
```

4. (15 marks) Construct an equivalent DFA for the following grammar

$$
\begin{array}{lll}
S & \longrightarrow & aA \\
S & \longrightarrow & bB \\
S & \longrightarrow & aS \\
S & \longrightarrow & baC \\
A & \longrightarrow & b \\
A & \longrightarrow & c \\
A & \longrightarrow & a
\end{array}
\qquad
\begin{array}{lll}
A & \longrightarrow & aB \\
B & \longrightarrow & bb \\
B & \longrightarrow & bC \\
C & \longrightarrow & bA \\
C & \longrightarrow & ba \\
C & \longrightarrow & a \\
C & \longrightarrow & abB \\
C & \longrightarrow & aC
\end{array}
$$

5. Prove that the following languages are context free:
   (a) (10 marks) $\{a^{n-2}b^{m+2}c^{n-1} \mid m, n \geq 2\}$
   (b) (10 marks) $\{a^{n+1}b^{n-1}c^{m+1}d^{m-2} \mid m, n \geq 2\}$
   (c) (10 marks) $\{a^{2n-1}c^2(bc)^{3n+1} \mid n \geq 4\}$
   (d) (10 marks) $\{uc^n v \mid 2|v|_a + 2|v|_b = 3|u|_a + |u|_b, n \geq 1\}$

6. (20 marks) Given the following grammar:

```
S -> aS | bS | cSa |aAaB |AbBbA
A -> baB | bC | a | c
B -> abA | bC |AC |a
C -> aA | baB | bbb  | a
```

   (a) (10 marks) Construct the PDA that accepts the same language by empty stack.
   (b) (10 marks) Construct an equivalent PDA that accepts the same language by final states.

7. Prove that the following languages are not context free:
   (a) (10 marks) $\{a^{p+4} \mid p \text{ is prime}, p > 7\}$
   (b) (10 marks) $\{a^{3n}b^{2n}c^n \mid n \geq 2\}$
   (c) (10 marks) $\{a^{n^3+3n^2+3} \mid n \geq 3\}$
   (d) (10 marks) $\{a^{2n}c^4b^{5n}c^{6n} \mid n \geq 0\}$

   The proofs must be correct to receive points.