Assignment #1

Date Due: May 20, 2022

Total: 100 marks

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1. (20 marks) Compute the languages

   $L_1 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ that begin with 0101$\}$

   $L_2 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ that end with 10111$\}$

   $L_3 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ that begin with 01001$\}$

   For each of the languages

   (a) $L_4 = L_1 \cap L_2$, and

   (b) $L_5 = L_3 \cap L_2$

   give a DFA for accepting it over the alphabet $\{0, 1, 2\}$.

2. (40 marks) Compute the following languages over the alphabet $\{0, 1, 2\}$:

   (a) the set of all strings consisting of alternating groups of 11 and 120 (11 and 120 *alternates* at least once);

   (b) the set of all strings whose fourth symbol from the right end is a 0;

   (c) the set of strings that either begin, or end (or both) with 1020;

   (d) the set of strings such that the number of 0's is divisible by six, and the number of 2's is not divisible by seven.

   and for each case give a DFA accepting it.

3. (20 marks) Give DFA's accepting the following languages over the alphabet $\Sigma = \{0, 1, 2, 3, 5\}$:

   (a) the set of all strings beginning with a 1, 3 or 5, that, when the string is interpreted as an integer in base 7, is a multiple of 6 plus 3. For example:

   - strings 3, 30, 555, 333, 50013, 50121, 33333, 5022, 50301, and 555552 are in the language;
   - the strings 20, 00, 022, 0020, 37 , 23, 5057, 223, 2325, 2375, 5, 32222, 505, 22, 72, and 035 are not.

(b) The set of all strings that ends with an **1, 3, or 5** and when the string is interpreted ***in reverse*** as an integer **in base 8, is a multiple of 6 plus 3**. A Examples of strings in the language are 3, 03, 555, 333, 31005, 12105, 33333, 2205, 10305, and 255555 Examples of strings that are not in the language are: 02, 00, 220, 0200, 73 , 32, 7505, 322, 5232, 5732, 5, 22223, 505, 22, 27, and 530.

4. (20 marks) Let $p \in \mathbb{N}$, p prime, $p > 4$, and $h \in \mathbb{N}$ such that $1 \le h < p$. We have a DFA $A = (\Sigma, Q, \delta, 0, F)$ with $Q = \{0, 1, \ldots, k\}$, $k \ge p$, $\{a, b, c\} \subseteq \Sigma$. We have that $\delta(q, a) = q + (p-2)h \bmod p$, for all states $q \in Q$. In these conditions:

   (a) show by induction on n that for all $n \ge 0$ and $q < p$, $\overline{\delta}(q, a^{n \cdot p}) = q$;

   (b) show that either $\{a^p\}^* baca^p abc \subseteq L(A)$, or $\{a^p\}^* baca^p abc \cap L(A) = \emptyset$.

5. (10 marks) Consider the DFA with the following transition table:

	0	1
→ 0	1	0
1	2	1
2	3	2
∗ 3	1	3

   Informally describe the language accepted by this DFA, and prove that your description is correct. You may use a proof based on induction on the length of an input string.

6. (10 marks) Repeat the above exercise for the following transition table:

	0	1
→ A	D	A
∗ B	C	B
C	B	C
∗ D	B	D

The maximum is bounded to 115 marks.

Very Important: Verify your solutions using Grail; describe ***how do you think*** for each of the above exercises. Just giving the final solution without any explanation may result in a mark of 0 at the discretion of your instructor.

If you decide for a late submission, please, contact me, before the due date, because I will give the solutions to ***all*** exercises in class.